How to calculate the share for the central pixel

The shape of the Gaussian curve is determined by the full width at half maximum, (see note 1)

$$\sigma = \frac{FWHM}{2.355}$$

Where the normalized Gaussian is

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$

Normalized in the sense that the area enclosed is 1, i.e.

$$\int_{-\infty}^{\infty} G(x) = 1$$

The error function is defined as the probability of finding the value between $-\frac{x}{\sigma\sqrt{2}}$ and $+\frac{x}{\sigma\sqrt{2}}$

$$\int_{-x}^{x} G(x) \cdot dx = Erf\left(\frac{x}{\sigma\sqrt{2}}\right)$$

Which is non integrable in terms of explicit functions, it can be approximated however, for small values of x by,

$$Erf(x) = \frac{2}{\sqrt{\pi}} \cdot \left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \ldots\right)$$

Thus the share for the central pixel can be written as

$$P = Erf\left(\frac{pixel}{2\sigma\sqrt{2}}\right)$$

$$Share \approx P^2$$

Examples :

1) FWHM = 4”, pixel = 0.71”

$\sigma = 4/2.355 = 1.698$

P = Erf(.148)
P = .1656
Share for central pixel = 0.027

2) FWHM = 4”, pixel = 2.5”

$\sigma = 4/2.355 = 1.698$
P = Erf(.52)
P = .54
Share = .29

3) FWHM = 4”, pixel = 2.7”

$\sigma = 4/2.355 = 1.698$
P = Erf(.56)
P = .57
Share = .33

Notes :

(1) The Full width at the half maximum

This is a property of the Gaussian distribution defining the curve's shape.

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$

for x=0 (the max) we have $G(0) = \frac{1}{\sigma\sqrt{2\pi}} = G\max$

Now we look for the value of x making G one half of this max value,

$$\frac{1}{2\sigma\sqrt{2\pi}} = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$

Simplifying and taking the logarithm we have for x

$$\frac{1}{2} = e^{-\frac{x^2}{2\sigma^2}}; \qquad \ln\left(\frac{1}{2}\right) = -\frac{x^2}{2\sigma^2}; \quad -\ln(2) = -\frac{x^2}{2\sigma^2}$$

$$x = \sigma \cdot \sqrt{2\ln 2}$$

The double of this value is what we call FWHM,

$$FWHM = 2\sigma \cdot \sqrt{2\ln 2}$$

$$FWHM = 2.354820 \cdot \sigma$$